MATH 718: RANDOMIZED LINEAR ALGEBRA AND APPLICATIONS

Fall 2024

COURSE INFORMATION

Instructor	Hanbaek Lyu	(Email: *hlyu@math.wisc.edu*, Office: Van Vleck 303)
Lectures	MWF 11:00AM - 11:50AM at Van Vleck B119	
Office hours	(tentative) T W 9-10 AM	
Canvas	https://canvas.wisc.edu/courses/427595	
Piazza	https://piazza.com/wisc/fall2024/math718	
Grader	Julia B. Nakhleh (Email: jnakhleh@wisc.edu)	

GENERAL INFORMATION

Instructional Modality In-person

Credits 3.0. The three credit hours are met by two 75-minute class periods and a for about 2-3 hours outside of class student work for every class period.

Regular and Substantive Student-Instructor Interaction

Participation in regularly scheduled learning sessions (where there is an opportunity for direct interaction between the student and the qualified instructor), office hours.

COURSE DESCRIPTION

Random solvers have been playing increasingly crucial roles in the modern computational tasks. The recent breakthroughs in applied and computational linear algebra that incorporate techniques of randomization have proven to be of great importance in modern applied math, computational sciences and data science, such as inverse problems, machine learning and scientific computing. The guiding principle is that one may greatly reduce computational and storage expenses at the cost of a small probability of failure.

This course will provide the students a systematic study of these modern methods of randomized linear algebra solvers, presenting mathematical backgrounds, algorithms, and concrete applications. Core theoretical topics include randomized Kaczmarz and its generalization to stochastic gradient descent, randomized singular value decomposition, random sketching, matrix completion, and compressive sensing, and corresponding applications.

COURSE MATERIALS

- Primary: Tropp, Randomized Algorithms for Matrix Computations
  Mahoney, Lecture Notes on Randomized Linear Algebra
- Preliminary:
  - probability: Wainwright, and Vershynin[Sec. 2, 5],
  - linear algebra (matrix factorization): Applied Numerical Linear Algebra, James Demmel[Sec. 3]
- Theory:

  - Randomized solvers: Randomized Kaczmarz Strohmer2007randomized[Sec. 2, 3] / Generalize to SGD Botou[Sec. 3, 4]
  - Randomized subspaces: Random SVD Halko[Sec. 1, 9, 10]
  - Randomized dimension reduction: random sketching Woodruff[Sec. 2]
  - Randomized reconstruction: matrix completion Recht-simplified, Candés (Need to prepare some simple presentation)
  - Compressive sensing (Lustig et al.)
- Applications:
  - Inverse problems and Scientific Computing
  - Image reconstruction; image denoising
  - Graph and Network Reconstruction
  - Supervised feature extraction (using supervised PCA and Matrix Factorization)

REQUISITES

Graduate/professional standing or member of the Pre-Masters Mathematics (Visiting International) Program.

COURSE LEARNING OUTCOMES

- Use or implement, in a computer language, a number of random linear algebra solvers, namely, matrix completion, randomized SVD, matrix sketching Kaczmarz algorithm
- Identify the conditions, assumptions, and the effectiveness of these algorithms
- Reproduce the proof strategies and summarize the main parts of them
- Apply these algorithms on a set of problems from image reconstruction, inverse problems and topic modeling
- Model problems from physical or social sciences into matrix form and identify regimes to apply the algorithms

WEEKLY SCHEDULE (TENTATIVE)

Week no.	material
1	Introduction, Randomized Kaczmarz
2	Concentration inequalities (random variables)
3	Concentration inequalities (random matrices)
4	Random sampling and applications (Matrix Monte Carlo)
5	Random sampling and applications (Matrix multiplication)
6	Random Sketching and applications (Gaussian and JL)
7	Random Sketching and applications (Gaussian and JL)
8	Random Sketching and appliations (Structured)
9	Randomized Least Squares
10	Randomized Least Squares
11	Randomized SVD
12	Randomized SVD
13	Applications (High probability MLE guarantee)
14	Inverse problems??

GRADING AND EXAMS

Grading Scheme	Homework (30%) + Midterm (30%) + Final (40%) (You should attend the final exam to pass the course.)
Midterm	Wednesday, 10/30, 5:45PM-7:15PM (VV B119)
Final	Saturday, 12/14, 2:45 - 4:45PM (Location TBD)

HOMEWORK

- Homework problems will be posted every other Thursday under "Assignments" in Canvas and will be due the following **Wed. 10 pm**. The lowest homework score will be dropped.
- Homework will be handed in, graded, and returned using Canvas. It should be uploaded to Canvas in the format of a **single PDF file**. (Latex typesetting is recommended.) Put your problems in the correct order (to simplify this, it might be useful to write each problem on a separate sheet of paper (certainly if you do not type your solutions). Please also make sure all pages are in the right orientation when you convert them. Do not hand in your rough draft or first attempt. Papers that are unreadable or disorganized cannot be graded.
- The homework is graded according to its correctness, completeness and presentation. Answers alone carry no credit. One should provide clear arguments and steps that lead to your solution/conclusion. Organize your work neatly. Use proper English. Answers should be simplified as much as possible.
- If you have mathematical questions about HW, I strongly encourage you to use Piazza (you may use anonymous option if you wish). You certainly can write me an e-mail with the questions, but Piazza is much more useful for mathematical formulas than e-mail programs.
- Observe rules of academic integrity. Handing in plagiarized work, whether copied from a fellow student or off the web, is not acceptable. Plagiarism cases will lead to sanctions. You can discuss the problems, ideas, hints with your classmates, however, you should always write down the solutions on your own.

CAMPUS SPACES FOR VIRTUAL LEARNING & TESTING

Dedicated on-campus spaces with high-speed internet are available for students to reserve (see LINK) for any exam/quiz taken during the semester. Computers can also be requested.

COURSE EVALUATION

UW-Madison now uses an online course evaluation survey tool, AEFIS. You will receive an official email two weeks prior to the end of the semester when your course evaluation is available. You will receive a link to log into the course evaluation with your NetID where you can complete the evaluation and submit it, anonymously. Your participation is an integral component of this course, and your feedback is important to me. I strongly encourage you to participate in the course evaluation. You are also very welcome to provide a feedback about the course during the semester (as a direct e-mail to me, or, if you prefer the anonymous option, you can use anonymous comments on Piazza).

ACADEMIC INTEGRITY

By enrolling in this course, each student assumes the responsibilities of an active participant in the UW-Madison community of scholars, in which everyones academic work and behavior are held to the highest academic integrity standards. Academic misconduct compromises the integrity of the university. Cheating, fabrication, plagiarism, unauthorized collaboration, and helping others commit these acts are examples of academic misconduct, which can result in disciplinary action. This includes but is not limited to failure on the assignment/course, disciplinary probation, or suspension. Substantial or repeated cases of misconduct will be forwarded to the Office of Student Conduct and Community Standards for additional review. For more information, refer to LINK.

ACADEMIC CALENDAR& RELIGIOUS OBSERVANCES

See LINK.

ACCOMMODATIONS FOR STUDENTS WITH DISABILITIES

The University of Wisconsin-Madison supports the right of all enrolled students to a full and equal educational opportunity. The Americans with Disabilities Act (ADA), Wisconsin State Statute (36.12), and UW-Madison policy (Faculty Document 1071) require that students with disabilities be reasonably accommodated in instruction and campus life. Reasonable accommodations for students with disabilities is a shared faculty and student responsibility. Students are expected to inform the instructor of their need for instructional accommodations by the end of the third week of the semester, or as soon as possible after a disability has been incurred or recognized. The instructor will work either directly with you or in coordination with the McBurney Center to identify and provide reasonable instructional accommodations. Disability information, including instructional accommodations as part of a student's educational record, is confidential and protected under FERPA. For more information, refer to LINK.

DIVERSITY AND INCLUSION

Diversity is a source of strength, creativity, and innovation for UW-Madison. We value the contributions of each person and respect the profound ways their identity, culture, background, experience, status, abilities, and opinion enrich the university community. We commit ourselves to the pursuit of excellence in teaching, research, outreach, and diversity as inextricably linked goals.

The University of Wisconsin-Madison fulfills its public mission by creating a welcoming and inclusive community for people from every background – people who as students, faculty, and staff serve Wisconsin and the world. For more information, refer to LINK.

RULES, RIGHTS & RESPONSIBILITIES

During the global COVID-19 pandemic, we must prioritize our collective health and safety to keep ourselves, our campus, and our community safe. As a university community, we must work together to prevent the spread of the virus and to promote the collective health and welfare of our campus and surrounding community. For more information, refer to LINK.

UW-MADISON BADGER PLEDGE

See LINK.

PRIVACY OF STUDENT INFORMATION & DIGITAL TOOLS: TEACHING & LEARNING ANALYTICS & PROCTORING STATEMENT

The privacy and security of faculty, staff and students personal information is a top priority for UW-Madison. The university carefully reviews and vets all campus-supported digital tools used to support teaching and learning, to help support success through learning analytic, and to enable proctoring capabilities. UW-Madison takes necessary steps to ensure that the providers of such tools prioritize proper handling of sensitive data in alignment with FERPA, industry standards and best practices. Under the Family Educational Rights and Privacy Act (FERPA which protects the privacy of student education records), student consent is not required for the university to share with school officials those student education records necessary for carrying out those university functions in which they have legitimate educational interest. 34 CFR 99.31(a)(1)(i)(B). FERPA specifically allows universities to designate vendors such as digital tool providers as school officials, and accordingly to share with them personally identifiable information from student education records if they perform appropriate services for the university and are subject to all applicable requirements governing the use, disclosure and protection of student data.

PRIVACY OF STUDENT RECORDS AND THE USAGE OF AUDIO RECORDED LECTURES

Lecture materials and recordings for this course are protected intellectual property at UW-Madison. Students in this course may use the materials and recordings for their personal use related to participation in this class. Students may also take notes solely for their personal use. If a lecture is not already recorded, you are not authorized to record my lectures without my permission unless you are considered by the university to be a qualified student with a disability requiring accommodation. [Regent Policy Document 4-1] Students may not copy or have lecture materials and recordings outside of class, including posting on internet sites or selling to commercial entities. Students are also prohibited from providing or selling their personal notes to anyone else or being paid for taking notes by any person or commercial firm without the instructors express written permission. Unauthorized use of these copyrighted lecture materials and recordings constitutes copyright infringement and may be addressed under the universitys policies, UWS Chapters 14 and 17, governing student academic and non-academic misconduct.

CAMPUS GUIDANCE ON THE USE OF FACE COVERINGS

Students of the class are expected to comply with the Universitys current COVID rules and policies that are maintained here: (see in particular link).

Face coverings must be correctly worn on campus at all times and in all places (both outside and inside), except by students in their assigned residence hall rooms; by employees when alone in a private, unshared lab or office; when traveling alone in a private vehicle; and when exercising outside in a way that maintains 6 feet of distance from other people.

Students with disabilities or medical conditions who are unable to wear a face covering should contact the McBurney Disability Resource Center or their Access Consultant if they are already affiliated. Students requesting an accommodation unrelated to disability or medical condition, should contact the Dean of Students Office.

Students who choose not to wear a face covering may not attend in-person classes, unless they are approved for an accommodation or exemption. All other students not wearing a face covering will be asked to put one on or leave the classroom. Students who refuse to wear face coverings appropriately or adhere to other stated requirements will be reported to the Office of Student Conduct and Community Standards and will not be allowed to return to the classroom until they agree to comply with the face covering policy. An instructor may cancel or suspend a course in-person meeting if a person is in the classroom without an approved face covering in position over their nose and mouth and refuses to immediately comply.